O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4503
Random Signals and Noise
Spring 2002

Midterm Exam #1

Graduate Students do all five problems,
others choose any four out of five.
Please specify below which four you choose to be graded.

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Name : _______________________________

Student ID: _________________________________

E-Mail Address:____

Problem 1:
Given that two events $\overline{A}_1$ and $\overline{A}_2$ are statistically independent, show that A_1 is also independent of A_2,
i.e., given $P(\overline{A}_1 \cap \overline{A}_2) = P(\overline{A}_1)P(\overline{A}_2)$, prove $P(A_1 \cap A_2) = P(A_1)P(A_2)$.

Problem 2:
A pharmaceutical product consists of 100 pills in a bottle. Two production lines used to produce the product are selected with probabilities 0.45 (line one) and 0.55 (line two). Each line can overfill or underfill bottles by at most 2 pills. Given that line one is observed, the probabilities are 0.02, 0.06, 0.88, 0.03 and 0.01 that the numbers of pills in a bottle will be 102, 101, 100, 99 and 98, respectively. For line two, the similar respective probabilities are 0.03, 0.08, 0.83, 0.04 and 0.02.

a) Find the probabilities that a bottle of the product will contain 99 pills and 102 pills.
b) Given that a bottle contains the correct number of pills, what is the probability it came from line two?
c) What is the probability that a purchaser of product will receive less than 100 pills in the bottle?

Problem 3:

A random variable X has the distribution function

$$F_X(x) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n).$$

a) Show if this is a valid distribution function.

b) If so, find the probabilities of $P(-\infty < X \leq 6.5)$, $P(X - 2 > 4)$, and $P(|X-5| > 3)$.

Problem 4:
The *Laplace* density function

$$f_X(x) = \frac{1}{2b} e^{-|x-m|/b}$$

has a characteristic function

$$\Phi_X(\omega) = \frac{e^{jm\omega}}{1+(b\omega)^2}\,.$$

Use this characteristic function to find the mean and variance of the random variable *X*.

Problem 5:
In a computer simulation, it is desired to transform numbers, that are values of a random variable uniformly distributed on (0,1), to numbers that are values of an exponential distributed random variables, as defined by

$$F_X(x) = \begin{cases} 1 - e^{-(x-a)/b}, & x > a \\ 0, & x < a \end{cases}$$

with $a = 0$. Find the required transformation.